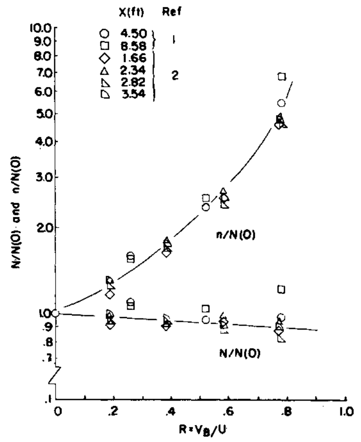

Fig. 2 Slopes of relative and absolute velocity profiles.

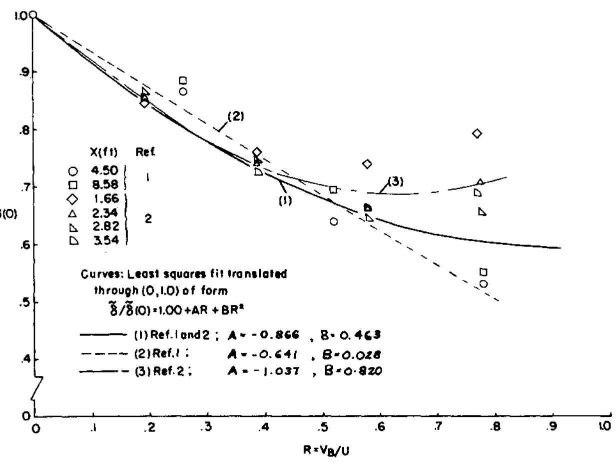

Fig. 3 Variation of boundary-layer thickness with ground plane speed.

Integral Parameters

The usual integral parameters can be developed using their respective definitions and Eq. (2).

$$\delta^*/\delta^*(0) = (1-R)\tilde{\delta}/\delta(0) \tag{3}$$

$$\theta/\theta(0) = (1-R)\left(\frac{N+2R}{N}\right)\tilde{\delta}/\delta(0) \tag{4}$$

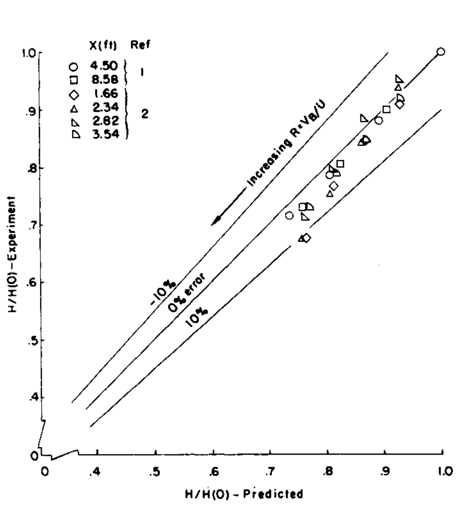

Fig. 4 Comparison of experimental and predicted values of shape parameter.

$$H/H(0) = N/(N+2R) \tag{5}$$

Once stationary ground plane parameters are computed, these equations can be used to predict the moving ground plane values, provided a suitable expression for $\tilde{\delta}/\delta(0)$ can be developed. A rather large amount of scatter is however apparent in empirical values of $\tilde{\delta}/\delta(0)$ beyond $R = 0.5$ (Fig. 3). Therefore only values of $H/H(0)$ are presented for comparison in this Note (Fig. 4).

Conclusions

Empirical information indicates that the velocity distribution in a turbulent boundary layer over a moving ground plane can be adequately represented by Eq. (2).

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Response of a Hot Wire Oscillating in a Shear Flow

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Introduction

A HOT wire is known to have a d.c. response proportional to the normal component of the mean flow velocity vector and an a.c. response proportional to the normal component of the instantaneous velocity fluctuation. In this paper the response of a hot wire oscillating in a shear flow is examined.

The theory is developed for a linearized constant temperature wire oscillating in a two-dimensional shear flow. By performing a Taylor series expansion in the direction of oscillation the wire response can be ordered in terms of the increasing harmonics and powers of the dimensionless amplitude of oscillation. Analysis of the harmonic components of the wire response in the one-dimensional case yields the first and second mean flow spatial derivatives in the direction of the wire oscillation. In the two-dimensional case the same analysis serves to separate the two components of the mean flow and also yields their first spatial derivatives in the direction of the wire oscillation. Separation of the harmonic components of the wire response which are coherent with the forced oscillation is achieved with a two phase lock-in amplifier.

This paper gives an account of the development of the theory and a discussion of the measurement technique. Recent experimental work[1,2] has established the validity of the technique in the one-dimensional case.

Theory

A shear flow is assumed to have three-dimensional mean velocity components U_1 (x_1, x_2, x_3), U_2 (x_1, x_2, x_3) and U_3

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(x_1, x_2, x_3) together with instantaneous turbulent fluctuations u_1 (x_1, x_2, x_3, t), u_2 (x_1, x_2, x_3, t) and u_3 (x_1, x_2, x_2, t). The hot wire, with its axis orientated perpendicular to the plane defined by U_1, U_2, performs simple harmonic motion in the x_2 direction about point P, Fig. 1. It is assumed that the wire is insensitive to any velocity component along its axis.

The magnitude of the instantaneous velocity Q at P is given by

$$Q_P = (\tilde{U}_1{}^2 + \tilde{U}_2{}^2 + \tilde{U}_3{}^2)^{1/2} \tag{1}$$

where

$$\tilde{U}_i = U_i + u_i; \quad i = 1, 2, 3 \tag{2}$$

The wire motion is described by

$$x_2 - x_{2P} = R \sin \omega t \tag{3}$$

Thus, the magnitude of the instantaneous velocity Q_w as seen by the wire is

$$Q_w = [\tilde{U}_1{}^2 + (\tilde{U}_2 + R\omega \cos \omega t)^2]^{1/2} \tag{4}$$

Expanding Eq. 4 yields

$$Q_w{}^2 = U_1{}^2 + 2u_1 U_1 + u_1{}^2 + U_2{}^2 + 2u_2 U_2 + u_2{}^2 + 2R\omega(U_2 + u_2)\cos \omega t + (\omega R)^2 \cos^2 \omega t \tag{5}$$

To facilitate the ordering of the terms in Eq. (5), introduce the following parameters: δ (x_1, x_3) a shear zone thickness in the x_2 direction, U_R a reference velocity equal to $(U_1{}^2 + U_2{}^2)^{1/2}$, $x_2{}^* = x_2/\delta$, and $\varepsilon = R/\delta \ll 1$. Express each term on the right of Eq. (5) in its Taylor series expansion about point P in the x_2 direction. After some algebraic manipulation Eq. (5) may be written in terms of increasing harmonics and powers of ε as follows:

$$Q_w{}^2 = U_R{}^2[F + G \sin \omega t + G^* \cos \omega t + H^* \sin 2\omega t + H \cos 2\omega t + \cdots] \tag{6}$$

The coefficients, accurate to order ε^2, are given by the following relations. The primes denote differentiation with respect to $x_2{}^*$.

$$F = \left(\frac{U_1 + u_1}{U_R}\right)^2 + \left(\frac{U_2 + u_2}{U_R}\right)^2 + \frac{\varepsilon^2}{U_R{}^2}\{\delta^2\omega^2 U_R{}^2 + U_1 U_1'' + U_2 U_2'' + (U_1')^2 + (U_2')^2 + U_1'' u_1 + 2u_1' U_1' + U_1 u_1'' + u_1 u_1'' + U_2'' u_2 + 2u_2' U_2' + U_2 u_2'' + u_2 u_2'' + (u_1')^2 + (u_2')^2\} + 0(\varepsilon^3, \varepsilon^4, \ldots) \tag{7}$$

$$G = (2\varepsilon/U_R{}^2)\{U_1 U_1' + U_1' u_1 + U_1 u_1' + u_1 u_1' + U_2 U_2' + U_2' u_2 + u_2' U_2 + u_2' u_2\} + 0(\varepsilon^3, \varepsilon^5, \ldots) \tag{8}$$

$$G^* = (2\varepsilon/U_R{}^2)(\delta\omega)(U_2 + u_2) + 0(\varepsilon^3, \varepsilon^5, \ldots) \tag{9}$$

$$H^* = (\varepsilon^2/U_R{}^2)(\delta\omega)(U_2' + u_2') + 0(\varepsilon^4, \varepsilon^6, \ldots) \tag{10}$$

$$H = (\varepsilon^2/2U_R{}^2)\{\delta^2\omega^2 U_R{}^2 - [U_1 U_1'' + U_2 U_2'' + (U_1')^2 + (U_2')^2 + U_1'' u_1 + 2u_1' U_1' + U_1 u_1'' + u_1 u_1'' + U_2'' u_2 + 2U_2' u_2' + U_2 u_2'' + u_2 u_2'' + (u_1)^2 + (u_2')^2]\} + 0(\varepsilon^4, \varepsilon^6, \ldots) \tag{11}$$

To process the hot wire anemometer signal a linearizer is employed as shown in Fig. 1; thus, it is desirable to expand Eq. (6) in a binomial series to obtain an explicit expression for Q_w. Maintaining accuracy in all terms to order ε^2, the binomial expression yields

$$Q_w = F^{1/2} U_R\left\{1 + \frac{1}{16F^2}(G^2 + G^{*2})\right\} + \frac{G}{2F}\left(1 + \frac{G^*}{4F}\right)\sin \omega t + \frac{G^*}{2F}\cos \omega t + \frac{H^*}{2F}\sin 2\omega t + \left[\frac{H}{2F} + \frac{1}{16F^2}(G^2 - G^{*2})\right] \times \cos 2\omega t + \cdots \tag{12}$$

The d.c. term and the harmonic coefficients in Eq. (12) will now be examined for the information they contain.

By passing the oscillating wire signal through the linearizer and an active filter, the d.c. component may be examined. Denoting a time mean by $\langle\ \rangle$, expanding the d.c. part of Eq. (12), and writing the result in terms of dimensional velocity yields

$$Q_{\text{d.c.}} = (U_1{}^2 + U_2{}^2)^{1/2}(1 + \tfrac{1}{2}\langle u_2{}^2\rangle/(U_1{}^2 + U_2{}^2)^{1/2} + \cdots) + 0(\varepsilon^2, \ldots) \tag{13}$$

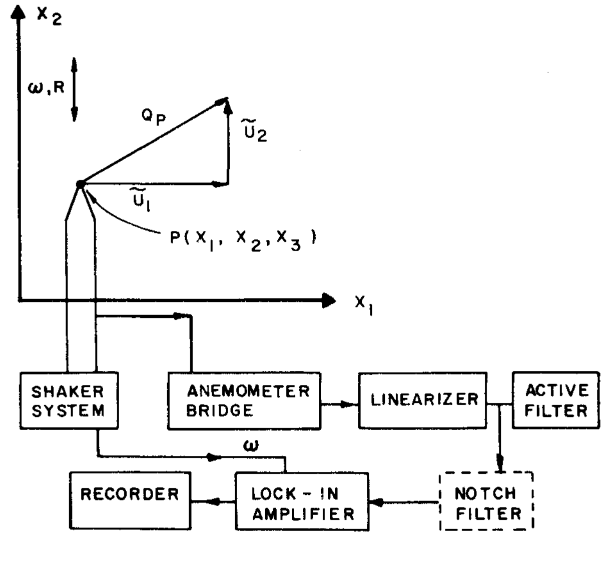

Fig. 1 Experimental schematic.

In the special case when $U_2 = 0$, Eq. (14) reduces to

$$Q_{\text{d.c.}} = U_1(1 + \tfrac{1}{2}\langle u_2{}^2\rangle/U_1 + \cdots) + 0(\varepsilon^2, \ldots) \tag{14}$$

Thus, in either Eq. (13) or (14) the d.c. term reduces to the linearized mean velocity[3] plus terms of order ε^2 and higher. If the turbulence level is low then Eqs. (13) and (14) reduce to $(U_1{}^2 + U_2{}^2)^{1/2}$ and U_1 respectively, provided the ε^2 terms are made negligible.

Passing the linearized signal directly into the two phase lock-in amplifier yields the in-phase and quadrature components of the hot wire signal coherent with the reference signal from the oscillation system as shown in Fig. 1.

Expanding the coefficient of $\cos \omega t$ in Eq. (12) yields

$$Q_{\cos \omega t} = [\varepsilon\delta U_2/(U_1{}^2 + U_2{}^2)^{1/2}] + 0(\varepsilon^3, \ldots) \tag{15}$$

The quantity $(U_1{}^2 + U_2{}^2)^{1/2}$ is known from the d.c. measurements. Thus, the combination of Eqs. (13) and (15) indicates that U_1 and U_2 can be measured from a single wire. In the case when $U_2 = 0$ the $\cos \omega t$ coefficient should be zero.

Expanding the coefficient of $\sin \omega t$ in Eq. (12) yields

$$Q_{\sin \omega t} = \frac{\varepsilon(U_1 U_1' + U_2 U_2')}{(U_1{}^2 + U_2{}^2)^{1/2}}\left[1 + \frac{\varepsilon\delta\omega U_2}{2(U_1{}^2 + U_2{}^2)}\right] + 0(\varepsilon^4, \ldots) \tag{16}$$

If the higher order terms are small enough to be neglected, Eq. (16) contains two unknowns, U_1' and U_2'. For the $U_2 = 0$ case Eq. (16) reduces directly to

$$Q_{\sin \omega t} = \varepsilon U_1' \tag{17}$$

By passing the linearized signal through the notch filter and into the two-phase lock-in amplifier, the $\sin 2\omega t$ and $\cos 2\omega t$ components can be isolated. Expanding the coefficient of $\sin 2\omega t$ in Eq. (12) yields

$$Q_{\sin 2\omega t} = (\varepsilon^2/2)[\delta\omega U_2'/(U_1{}^2 + U_2{}^2)^{1/2}] + 0(\varepsilon^4, \ldots) \tag{18}$$

Thus U_2' can be measured and from Eq. (16) U_1' is known.

In the special case when U_2 vanishes everywhere in the shear zone expanding the coefficient of $\cos 2\omega t$ in Eq. (12) yields

$$Q_{\cos 2\omega t} = (\varepsilon^2/4)(\delta^2\omega^2/U_1 - U_1'') + 0(\varepsilon^4, \ldots) \tag{19}$$

which provides a direct measurement of U_1''. When U_2 is not zero both U_1'' and U_2'' remain as unknowns in the expansion.

Preliminary experimental work[1,2] has shown the technique to be successful for one-dimensional flows. The application to a two-dimensional flow remains to be carried out.

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Nonlinear Flexural Vibrations of a Rotating Myklestad Beam

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Introduction

THE equations of motion of a rotating cantilevered blade vibrating in a plane other than the one perpendicular to the plane of rotation are nonlinear because of the couplings due to Coriolis forces. Carnegie[1] used a variational approach to derive the partial differential equations of motion of a pretwisted cantilevered blade mounted on the periphery of a rotating disk. His equations include the effects of rotatory inertia, torsion, bending and Coriolis accelerations, but are complicated and only solvable for very special cases. Rao and Carnegie[2] considered a uniform, untwisted, rotating cantilevered blade vibrating in its plane of rotation and simplified the problem considerably by ignoring the shear deformation and rotatory inertia effects. The fundamental nonlinear mode shape was obtained by the Ritz method. The purpose of this Note is to generate the nonlinear modes of a nonuniform rotating blade performing inplane vibrations by resorting to a Myklestad beam which is referred to here as a discrete model described in Ref. 3.

Analysis

Consider a nonuniform, untwisted, radial, cantilevered blade with a vertical plane of symmetry, mounted on the periphery of a disk rotating with a uniform angular velocity Ω about its polar axis. The Myklestad beam model of the blade, as shown in Fig. 1, has its inertia properties concentrated at n discrete mass stations along its axis, the beam sections between consecutive stations being massless but possessing bending and shear flexibilities. Each station i has a mass m_i and a bending inertia J_i located at a distance x_i from the axis of rotation. The section length from station i to station $i+1$ is l_i. The beam axis is assumed inextensional and vibration is restricted to the plane of rotation. Amplitudes are taken to be moderately large and therefore any deviation from the linear modes of the system would be small.

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Index category: Structural Dynamic Analysis.

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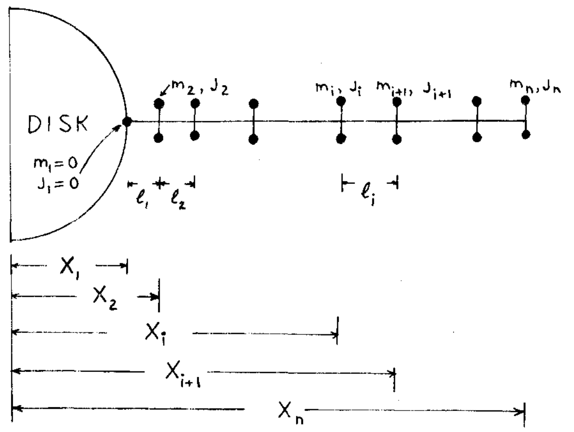

Fig. 1 Myklestad beam model of blade.

Using a two-mode approximation, we can write down expressions for the lateral displacement y_i and the beam slope α_i at the ith mass station in terms of the normal modes of the linear rotating beam as follows:

$$y_i = \sum_{j=1}^{2} A_i^{(j)} q_j(t), \qquad \alpha_i = \sum_{j=1}^{2} C_i^{(j)} q_j(t) \tag{1}$$

where $A_i^{(j)}$ is the ith component of the jth normal mode $\{A\}^{(j)}$, $C_i^{(j)}$ is the ith component of the associated slope vector $\{C\}^{(j)}$, and the $q_j(t)$ are normal coordinates. Including the contribution due to the centrifugal force field, the potential energy of the rotating Myklestad beam can now be written as[4]

$$V = (1/2) \sum_{j=1}^{2} \omega_{(j)}^2 M_{(j)} q_j^2 \tag{2}$$

where $\omega_{(j)}$ is the jth natural frequency of the linear rotating beam, and

$$M_{(j)} = \sum_{i=1}^{n} (m_i A_i^{(j)2} + J_i C_i^{(j)2}) \tag{3}$$

is the associated generalized mass.[4]

The potential energy expression given by Eq. (2) includes the effect of the centrifugal force field that acts upon the blade, which must now be considered nonrotating for the purpose of computing its kinetic energy. This kinetic energy should not have the effect of the centrifugal force field in it and is obtained by eliminating all terms involving Ω^2. However, all the nonlinear coupling terms must be retained. Denoting time differentiations by dots, the following kinetic energy expression is obtained after elimination of Ω^2 terms.[4]

$$T = (1/2) \sum_{i=1}^{n} \left[m_i \left\{ \dot{y}_i^2 + 2\dot{y}_i \Omega x_i + 2\Omega \left(y_i \sum_{j=1}^{i-1} l_j \alpha_j \dot{\alpha}_j - (1/2)\dot{y}_i \sum_{j=1}^{i-1} l_j \alpha_j^2 \right) \right\} + J_i \{ \dot{\alpha}_i^2 + 2\Omega \dot{\alpha}_i \} \right] \tag{4}$$

It can be shown[4] that using a one-mode approximation for y_i and α_i leads to an elimination of the problem nonlinearities. With the two-term approximation expressed in Eqs. (1) however, the Lagrangian equations of motion of the rotating Myklestad beam are two nonlinear coupled ordinary differential equations[4] in the normal coordinates $q_1(t)$ and $q_2(t)$. We are interested in a harmonic solution of the form

$$q_1 = B_1 \sin \omega t, \qquad q_2 = B_2 \sin 2\omega t \tag{5}$$

where B_1 and B_2 are arbitrary parameters and ω is the nonlinear frequency. Defining the average Lagrangian $\bar{L}$ as the time integral of the Lagrangian function taken over a period, and using Eqs. (1–5), the Ritz averaging method[2,4] ($\partial \bar{L}/\partial B_1 = 0$ and $\partial \bar{L}/\partial B_2 = 0$) yields the following relationships between the coefficients B_1 and B_2, the frequency ω and the other system parameters, for the fundamental nonlinear mode: